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The target space dependence of the Hagedorn temperature

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Abstract: The effect of certain simple backgrounds on the Hagedorn temperature in theories of closed strings is examined. The background of interest are a constant Neveu-Schwarz B-field, a constant offset of the space-time metric and a compactified spatial dimension. We find that the Hagedorn temperature of string theory depends on the parameters of the background. We comment on an interesting non-extensive feature of the Hagedorn transition, including a subtlety with decoupling of closed strings in the NCOS limit of open string theory and on the large radius limit of discrete light-cone quantized closed strings.

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One of the most interesting and general features of string theory is its exponentially increasing density of states [1, 2]. If one considers an ensemble of weakly interacting strings at finite temperature, this behavior of the density of states is thought to lead either to a limiting temperature or a phase transition. The limiting temperature is called the Hagedorn temperature.

In weakly coupled string theory this phenomenon can be understood in terms of how the density of states in a multi-string system depends on the energy. Below a certain energy scale, the dominant contribution to the density of states in a system of many strings is a thermal distribution of multi-string states. At a higher energy the statistically most likely configuration changes from this thermal distribution to one which is dominated by a single long string. This leads to an interesting non-extensive behavior of the thermodynamics at that point.

Recently, there has been some interest in the Hagedorn transition in background fields [3], particularly the behavior of open strings in the limit which produces non-commutative open string (NCOS) theory [4]–[8]. The phase diagram of these systems has an exceedingly rich structure [8]. It also has interesting analogs in gauge theory systems as was pointed out in a recent work [9].

One advantage of the NCOS limit is that closed strings, and therefore gravity, decouple [10, 11] from the open string degrees of freedom. This avoids problems which are expected to be inherent in trying to make a thermal ensemble in a theory of quantum gravity. Such a theory should suffer from the Jeans instability at finite temperature — which simply means that hot flat space is unstable, with the preferred state likely to be one where the energy density has collapsed into black holes whose Bekenstein-Hawking entropy is much greater than any ordinary particle states. It was argued in [12] that gravitational instability would make the Hagedorn transition of ordinary string theories into a first order transition and that it should actually occur at a temperature which is less than the Hagedorn temperature. On the other hand, it was argued in [4] that in the NCOS theory, since gravity decouples, the transition is of second order and can be studied in the context of weakly coupled string theory.

It was shown in [13] that when the space is compactified, wrapped states of closed strings do not decouple in the NCOS limit. These closed string states were used to construct wound string theory in [14] and non-relativistic closed strings [15]. One would expect that, in the limit as the compactification radius is large, the wrapped closed strings would couple more and more weakly and in the infinite, de-compactified limit they would disappear from the spectrum [16]. Indeed their energies do go to infinity. However, we shall show in this paper that their Hagedorn temperature remains, that is, no matter how large that radius is, they still participate in the Hagedorn transition. This means that they should make a contribution to the thermodynamic properties of the system.

We will begin by examining the effect of certain simple background fields on the Hagedorn temperature in theories of closed strings. The NCOS limit is accessible within this family of backgrounds and can be studied there. This background is a space-time with one compact dimension,

$$X^1 \sim X^1 + 2\pi R \tag{1}$$

a Neveu-Schwarz B-field and space-time metric of the form

$$G_{\mu\nu} = \begin{pmatrix} -1+A^2 & -A & 0 & \dots \\ -A & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}, \quad B_{\mu\nu} = \begin{pmatrix} 0 & B & 0 & \dots \\ -B & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}, \tag{2}$$

where A and B are constants. When $g_s = 0$, closed string theory is exactly solvable on this background. Ordinarily, closed strings do not couple to a constant B-field since, in the absence of D-branes it is gauge equivalent to a constant electromagnetic field and closed strings do not carry electromagnetic charges. Furthermore, they would not couple to A since it can be removed by a coordinate transformation. However, when the coordinate X^1 is compactified, neither the gauge transformation nor the coordinate transformation are compatible with the identification (1).

When a spatial dimension is compactified, the wrapped modes of closed strings are indeed affected by B which shifts their energy by a constant. The shift of energy of the wrapped states can be understood by considering a process where you make an wrapped closed string by transporting the ends of an open string around the compact dimension and then fusing them together. Then, transporting the charged endpoints of the open string in a constant B-field involves precisely the energy shift which produces the chemical potential for the resulting wrapped closed string state.

Similarly, on an un-compactified space, the parameter A can be shifted away by a re-definition of the coordinates,

$$G_{\mu\nu} dX^\mu dX^\nu = -dX^0 dX^0 + (dX^1 - A dX^0)(dX^1 - A dX^0) + \cdots . \tag{3}$$

However, when X^1 is periodically identified, this re-definition is not a symmetry of the space-time. In this case, the spectrum of closed strings also couples to A which shifts their energy by their momenta in the 1-direction. For example, a single bosonic closed string which wraps the compactified direction p times and which has l quanta of momentum in that direction has energy spectrum

$$P_0 = -\frac{BRp}{\alpha'} - \frac{Al}{R} + \sqrt{\left(\frac{pR}{\alpha'}\right)^2 + \left(\frac{l}{R}\right)^2 + \vec{P}^{\,2} + \frac{2}{\alpha'}(N + \tilde{N} - 2)} . \tag{4}$$

(Here $N = \sum_{n=1}^\infty \alpha_{-n} \cdot \alpha_n$ and $\tilde{N} = \sum_{n=1}^\infty \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n$ with standard notation for oscillators following [17] and $\vec{P}^{\,2} = P_i P^i$ with $i = 2, \dots, D-1$. A similar formula for type II superstrings is a straightforward generalization of (4). It should be supplemented by the appropriate level matching condition and, for the fermionic string, the GSO projection.)

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Our central result is that in the presence of A and B in the compact space, the Hagedorn temperature is modified to be

$$T_H = T_H^0 \sqrt{(1-A^2)(1-B^2)}\,, \tag{5}$$

where T_H^0 is the Hagedorn temperature of the string theory in the limit where $A = B = 0$. For the bosonic string $T_H^0 = 1/4\pi\sqrt{\alpha'}$ whereas it is $1/2\pi\sqrt{\alpha'}$ for the type II superstring.

The formula (5) has a remarkable feature. As expected, it depends on A and B. However, for fixed A and B, it does not depend on the compactification radius R. This is surprising for the following reason. The main role of A and B in the string spectrum is as chemical potentials for discrete momentum and wrapping modes respectively, as can be seen for example from the closed Bosonic string spectrum (4).

There is a region of the parameter space where A and B are between 0 and 1, away from their limiting values and where R is very large so that all wrapped states have a very large energy. In that case, at temperatures just below T_H, practically no wrapped states are excited in the thermal distribution. However, since T_H depends on B, it must be wrapped states which condense at the Hagedorn transition, in fact the resulting long string must wrap the compact dimension. An unwrapped long string could only become important at the higher temperature $T_H^0\sqrt{1-A^2} > T_H^0\sqrt{(1-A^2)(1-B^2)}$. Thus we see that, in the limit where R is very large, when the temperature T_H is reached, there is a catastrophic process where dominant configurations in the ensemble go from a thermal distribution of multi-string states with zero wrapping to a single long string which wraps the compact dimension.

In a thermal ensemble where the total energy is proportional to the volume, there is certainly sufficient energy to produce such a long string whose energy only scales like its length. Then, the R-dependence of the total energy, which grows linearly in R if the temperature is held fixed as R is changed, is similar to the energy dependence of a wrapped string which also scales linearly with R.

In [13] it was noted that, when the compactified dimension has finite radius, the wrapped closed string states do not decouple in the NCOS limit. These wrapped states should get infinitely large energy in the limit where the radius of the compact dimension is taken to infinity. However, we see that, no matter how large that radius is, the closed strings still participate in the Hagedorn transition. The phase transition of open strings in the decompactified NCOS limit is thought to be of second order [4]. We see that, if the radius is very large but finite, the closed string Hagedorn behavior makes it a first order transition.

It is clear from (5) that there are limiting values of both background fields A and B. The critical value of B is where the NCOS limit is found. A whole family of NCOS limits should arise in our model by changing A within its limiting values. As can be easily seen, a T-duality transformation along the compactified direction [18, 19], simply interchanges the role of B and A, $B \leftrightarrow A$. Since the Hagedorn temperature (5) is symmetric under this interchange, it is self-dual.

The limiting value of A is similarly interpreted as the DLCQ limit of the closed string theory. In fact, it can be seen explicitly that taking $A = 1$ in (4) (with the appropriate rescaling of R) reproduces the discrete light-cone quantization (DLCQ) spectrum of closed strings in a B-field that was discussed in [20]. There, it had an interesting interpretation in terms of covers of a torus that are expected to be found in the weak coupling limit of the matrix model of M-theory [21, 22]. The result of the present paper implies a curious non-decoupling in the DLCQ limit of closed strings. This is another limit of string theory which is described by a gauge theory, the matrix model, which does not involve gravity. We found in [20] that the B-field couples to the thermodynamic partition function of both free type II superstring theory and the matrix string. Indeed, the Hagedorn temperature there is also modified by a factor of $\sqrt{1-B^2}$, with no reference to the light-cone radius R. This poses a subtlety for discrete light-cone quantization of strings.

The energy spectrum in (4) is straightforward to obtain from canonical quantization of the string. The nature of the high energy density of states with such a spectrum was discussed in detail [23]. In fact there are several ways of finding the Hagedorn temperature. One is to estimate the asymptotic density of states $\rho(E) \sim \exp(\beta_H E)$ and find the coefficient in the exponential $\beta_H = 1/k_B T_H$ where k_B is the Boltzmann constant. In this paper we are using units where $k_B = 1$. Another [12] is to examine the spectrum of the string theory and see where a new tachyonic state appears. In all known cases, this temperature coincides with the Hagedorn temperature. Finally, the Hagedorn temperature can be defined as that temperature where, in euclidean space, the vertex operator

$$e^{2\pi i T X^0}$$

becomes a relevant operator.

It is this last criterion where the Hagedorn transition is seen to be analogous to the Berezinsky-Kosterlitz-Thouless (BKT) transition in the 2-dimensional X-Y-model, a parallel which has been drawn many times in the literature [24, 25]. In fact our present model could have an interesting analog in coupled X-Y-models where the metric and B-field couple the two angular degrees of freedom,

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \partial_a X^\mu \left(G_{\mu\nu}\delta^{ab} - B_{\mu\nu}\epsilon^{ab} \right) \partial_b X^\nu$$

The BKT transition involves the condensation of vortices. It is easy to see that the transition temperature is modified by A and B in the same way as the Hagedorn temperature.[1] The analog of the catastrophic behavior which we discussed at the Hagedorn temperature is a condensation of vortices of one of the variables X^0 induced by the B-coupling to X^1 in a state where the density of these vortices was zero just before the transition. It is possible that this process could be experimentally visible in Josephson junction arrays.[2]

Derivation of T_H: The free energy of a gas of relativistic Bose particles is

$$F = \frac{1}{\beta} \mathrm{Tr} \ln \left(1 - e^{-\beta P_0}\right) = -\sum_{n=1}^{\infty} \frac{1}{n\beta} \mathrm{Tr} e^{-n\beta P_0} . \tag{6}$$

Equation (6), can be used to derive the bosonic string free energy at one loop, by the standard procedure of computing the sum of free energies of the particles in the string spectrum. Canonical quantization of the string in the light-cone gauge give the energy spectrum (4) together with the level matching condition $\tilde{N} - N = pl$. To obtain the free energy of the bosonic string we use the integral identity

$$\int_0^{\infty} dt e^{-xt^2 - y/t^2} = \frac{1}{2}\sqrt{\frac{\pi}{x}} e^{-2\sqrt{xy}} ,$$

where

$$t^2 = 1/\tau_2 \,, \qquad x = \frac{n^2\beta^2}{4\pi\alpha'} \,, \qquad y = \pi\alpha' \left(\frac{R^2 p^2}{\alpha'^2} + \frac{l^2}{R^2} + \vec{P}^2\right) .$$

We also enforce the level matching condition with a Lagrange multiplier τ_1 to obtain the free energy of the bosonic string

$$\begin{aligned} F = -\sum_{n,p,l} \int_0^{\infty} \frac{d\tau_2}{2\tau_2} \int_{-1/2}^{1/2} \frac{d\tau_1}{(4\pi^2\alpha'\tau_2)^{13}} \left(\frac{\alpha'\tau_2}{R^2}\right)^{1/2} |\eta(\tau)|^{-48} \times \\ \times \exp\left[-\frac{\beta^2 n^2}{4\pi\alpha'\tau_2} - \pi\alpha'\tau_2 \left(\frac{l^2}{R^2} + \frac{R^2 p^2}{\alpha'^2}\right) - 2\pi i \tau_1 pl + n\beta B \frac{Rp}{\alpha'} + n\beta A \frac{l}{R}\right] . \end{aligned} \tag{7}$$

The temperature independent $n = 0$ term gives the vacuum energy, i.e. the cosmological constant contribution, the other terms give the relevant thermodynamic potential.

To perform the integration over τ_1 it is useful to rewrite the Dedekind eta function in terms of a series as in [17]. One has

$$|\eta(\tau)|^{-48} = e^{4\pi\tau_2} \left| \prod_{m=1}^{\infty} \left(1 - e^{2\pi i \tau m}\right) \right|^{-48} \tag{8}$$

[1] Note that the temperature at which the BKT transition occurs is not universal. Here, by BKT temperature, we mean the temperature at the zero coupling limit of the line of critical points.

[2] We thank Professor P. Sodano for discussions on this point.

and

$$\prod_{d=1}^{\infty}\left(1-z^{d}\right)^{-24} \equiv \sum_{r=0}^{\infty} d(r) z^{r}\,, \tag{9}$$

where $z=\exp(2\pi i\tau)$.

Using (8) and (9), the τ_1 integral in (7) can be easily performed

$$\sum_{r,r'=0}^{\infty} d(r)d(r')e^{-2\pi\tau_2(r+r')}\int_{-1/2}^{1/2} d\tau_1 e^{2\pi i\tau_1(r-r'+pl)} = \sum_{r=0}^{\infty} d(r)d(r+pl)e^{-2\pi\tau_2(2r+pl)}\,. \tag{10}$$

The coefficient $d(r)$ is given by

$$d(r)=\frac{1}{2\pi i}\oint\frac{G(z)}{z^{r+1}}\,, \tag{11}$$

where

$$G(z)\equiv\sum_{r=0}^{\infty} d(r)z^{r} = \mathrm{Tr}z^{N} = \prod_{r=1}^{\infty}\left(1-z^{r}\right)^{-24}\,. \tag{12}$$

The generating function $G(z)$ vanishes rapidly for $z\to 1$, while if r is very large z^{r+1} is very small for $z<1$. Consequently, for large r there is a sharply defined saddle point for z near 1. Following [2] one gets

$$d(r)\sim r^{-27/4}e^{4\pi\sqrt{r}}\,. \tag{13}$$

In the $\tau_2\to 0$ limit the sums are dominated by those integers for which r, l and p are such that r and $r+pl$ are big, so that (13) could be used for $d(r+pl)$. Moreover, the dominant term is obtained by setting $n=1$. Then for $\tau_2\sim 0$ we could use a saddle point procedure for the variables r, l and p to evaluate the sums

$$\sum_{l,p}\sum_{r=0}^{\infty} r^{-27/4}(r+pl)^{-27/4}e^{4\pi\left(\sqrt{r}+\sqrt{r+pl}\right)-2\pi\tau_2(2r+pl)-\pi\alpha'\tau_2\left(\frac{l^2}{R^2}+\frac{R^2p^2}{\alpha'^2}\right)+\beta B\frac{Rp}{\alpha'}+\beta A\frac{l}{R}}\,. \tag{14}$$

The equations that are to be solved to find the maximum of the exponent are

$$\begin{aligned}
\frac{1}{\sqrt{r}}+\frac{1}{\sqrt{r+pl}}-2\tau_2 &= 0\\
\beta\frac{A}{R}-2\pi\alpha'\tau_2\frac{l}{R^2}+2\pi\frac{p}{\sqrt{r+pl}}-2\pi\tau_2 p &= 0\\
\beta\frac{BR}{\alpha'}-2\pi\alpha'\tau_2\frac{pR^2}{\alpha'^2}+2\pi\frac{l}{\sqrt{r+pl}}-2\pi\tau_2 l &= 0\,.
\end{aligned} \tag{15}$$

The solutions for p, l and r read

$$\begin{aligned}
p &= \frac{\tau_2 r}{1-2\tau_2\sqrt{r}}\frac{\beta}{2\pi R}\left(\frac{A}{\tau_2\sqrt{r}}-A-B\right)\\
l &= \frac{\tau_2 r}{1-2\tau_2\sqrt{r}}\frac{\beta R}{2\pi\alpha'}\left(\frac{B}{\tau_2\sqrt{r}}-B-A\right)\\
\sqrt{r} &= \frac{1}{2\tau_2}\left(1\pm\sqrt{\frac{1+\frac{\beta^2}{16\pi^2\alpha'}(A-B)^2}{1+\frac{\beta^2}{16\pi^2\alpha'}(A+B)^2}}\right)\,.
\end{aligned} \tag{16}$$

To obtain the well-known solution for $A = B = 0$, i.e. $\sqrt{r} = 1/\tau_2$, we must choose the + sign in the last equation.

Substituting the solutions (16) in (7) the exponent becomes

$$\frac{2\pi}{\tau_2}\left\{1 - \frac{\beta^2}{8\pi^2\alpha'} - \frac{\beta^2(A^2+B^2)}{16\pi^2\alpha'} + \sqrt{\left(1 - \frac{\beta^2(A+B)^2}{16\pi^2\alpha'}\right)\left(1 - \frac{\beta^2(A-B)^2}{16\pi^2\alpha'}\right)}\right\}. \tag{17}$$

It is not difficult to see that this exponent vanishes when $T = T_H = 1/\beta_H$, where

$$T_H = \frac{\sqrt{(1-A^2)(1-B^2)}}{4\pi\sqrt{\alpha'}}. \tag{18}$$

The Hagedorn temperature does not depend on the compactification radius and is smaller then the Hagedorn temperature in the absence of A and B. It is interesting to notice that A and B play the role of the chemical potentials for the quantized momenta and winding modes in the compactified direction, respectively. In fact, the formula for the chemical potential dependent Hagedorn temperature derived in [23] can be shown to be identical to (18). In [23] the reduced chemical potentials for the quantized momenta and winding modes $\bar{\mu} = \beta\mu$ and $\bar{\nu} = \beta\nu$ were used. Thus, performing the necessary rescaling by β, one arrives, at the Hagedorn temperature

$$T_H = \frac{1}{4\pi\sqrt{\alpha'}}\sqrt{(1-(\nu R)^2)\left(1 - \left(\frac{\mu\alpha'}{R}\right)^2\right)}. \tag{19}$$

Comparing this with(18) we can identify

$$\nu \equiv \frac{A}{R}, \qquad \mu \equiv \frac{RB}{\alpha'}.$$

In terms of A and B any dependence on the compactification radius R disappears. This independence on R is remarkable since it must hold even if the compactification radius is arbitrarily large. Of course, without the compactification in the first place, T_H would be independent of A and B and would be the usual closed string value $1/4\pi\sqrt{\alpha'}$. This non-commutativity of compactifying and going to the Hagedorn temperature is a result of the exponential growth of the density of states of the string which is independent of compactification radius. At the Hagedorn temperature, the thermal distribution of string states is unstable and the most favorable configuration is one long string that contains all of the energy. In order to know about the A and B fields, this long string must wrap the compactified light-like direction. Because of this non-extensive behavior, it always has enough energy to do that, no matter how large the radius R.

Similar expressions with similar conclusions can be reached for the case of the type II superstring and the result is given in (5).

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